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Application of a permutation group on sasirangan pattern

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ABSTRACT

A permutation group is a group of all permutations of some set. If the set that forms a permutation group is the n-first of natural number, then a permutation group is called a symmetry group. There is another type of group, i.e., a cyclic group and a dihedral group, and they are a subgroup of a symmetry group by numbering the vertices of the polygon. Sasirangan is the traditional batik from the South Kalimantan. There are 18 traditional patterns. All the patterns make some polygon. Because of this, the purpose of this research is to investigate the type of group that forms the patterns of Sasirangan. First, the authors give the procedure to investigate the patterns of Sasirangan, then use that procedure to the patterns of Sasirangan. The result of this research is the patterns of Sasirangan form cyclic groups C_1 and C_2, and dihedral groups D_2, D_4, D_5 and D_8.

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INTRODUCTION

A group G is a nonempty set with a binary operation $\cdot : G \times G \rightarrow G$ that satisfies associative properties, there is an identity element in G, and every element in G has inverse under operation $\cdot$. The nonempty subset S of a group G is called a subgroup of G if S is closed under the same binary operation in G and the inverse element of S is also in S. Let $X = \{a_1, a_2, \ldots, a_n\} \subseteq G$. The set

$$\langle X \rangle = \{a_1^{t_1} a_2^{t_2} \ldots a_n^{t_n} \mid t_i = \pm 1, i = 1,2, \ldots, n\}$$

is the smallest subgroup in G that contains X. If $G = \langle X \rangle$, then G is called a group with a generator X (Malik et al., 1997). By this definition, the authors define a cyclic group in the following definition.

Definition 1 (Malik et al., 1997)
Let G be a group and $a \in G$. If $G = \langle a \rangle$, then G is called a cyclic group.

Theorem 1 (Malik et al., 1997)
If G is a finite cyclic group with $|G| = n$ then $G = \{ e, a, a^2, \ldots a^{n-1}\}$. Furthermore, the finite cyclic group with order n denoted as C_n.

Let X be a nonempty set. A permutation π of X is an injective function from X to X. The set of all permutations of X form a group given as the following definition.

Definition 2 (Malik et al., 1997)
Let X be a nonempty set. The permutation group S_X is the group of all permutations of X under a composition function operation.

There is a connection between any group with the permutation group. It's called Cayley's Theorem that given as follows.

Theorem 2 (Fraleigh, 2014)
Every group is isomorphic to a permutation group.

Let $I_n = \{1, 2, \dots, n\}$, if $X = I_n$ then S_X is called a symmetric group, denoted by S_n, defined as follows.

Definition 3 (Malik et al., 1997)
Let $I_n = \{1, 2, \dots, n\}$, $n > 1$ and let S_n be a set of all permutations on I_n. The permutation group $(S_n, \circ)$ is *called a symmetric group*.

A figure has rotational symmetry if it looks the same after rotating a certain number of degrees around a point. Furthermore, a figure has reflectional symmetry if it looks the same after reflecting it over a line, and the line is called the line of symmetry (Herrmann & Sally Jr., 2013). The number of times a figure fits onto itself in one complete rotational is called the order of rotational symmetry. The number of the line of symmetry is called the order of line symmetry. For example, i.e., an equilateral triangle has three rotational symmetries dan three-line symmetries shown in Figure 1.

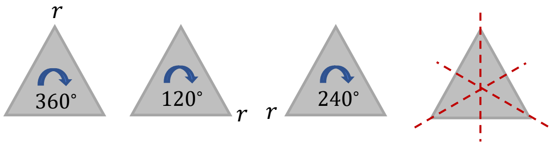


Figure 1. Rotational Symmetries and Line Symmetries of Equilateral Triangle

The symmetric group S_3 can be illustrated geometrically as the rotation and reflection of an equilateral triangle. There is another group that can be illustrated geometrically, i.e., a dihedral group

$$D_4 = \{\langle a, b \rangle \mid a^4 = b^2 = (ab)^2 = e\}.$$

If $a = (1\ 2\ 3\ 4)$ and $b = (1\ 2)$ then D_4 is a subgroup of S_4. The dihedral group D_4 can be illustrated as the rotation and reflection of squares (Gallian, 2017). In general, the dihedral group is defined as the following definition.

Definition 4 (Gallian, 2017)
A dihedral group of order $2n$, denoted as D_n $n \geq 3$, is a group with 2n generator that generated by two-element a and b satisfying

$$a^n = b^2 = (ab)^2 = e.$$

A dihedral group of order $2n$ is often called the group of symmetries of a regular n-gon (Gallian, 2017).

The symmetric group and the dihedral group have been applied in various fields, which can be seen in Crans et al. (2009), Viana (2015), Deloach (2020), Waseem et al. (2020), and Ghayoumi & Bansal (2017). There are also some research about the dihedral group namely, Al-Hasanat et al. (2014), (Vinod, 2021), and Dianat & Mogharrab (2019).

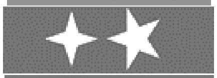
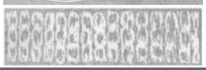
Sasirangan is a type of traditional batik cloth from South Kalimantan. Sasirangan has a distinctive pattern and shows the identity and behavior of the people of South Kalimantan. Therefore, knowing and understanding the history of the values in the Sasirangan patterns makes one understand the culture and behavior of the people of South Kalimantan (Almas, 2018). Based on this, the authors researched the application of group permutation to the Sasirangan pattern.

METHOD

This research begins with constructing steps to identify the type of group based on Definition 1, Definition 3, and Definition 4 on the pattern of the Sasirangan cloth. These steps are then applied to the Sasirangan pattern to determine the type of group formed.

There are 18 *Sasirangan* patterns stated in Table 1.

Table 1. Sasirangan Pattern

No.	Name of Pattern	Figure of Pattern
1	Gigi Haruan	
2	Kambang Kacang	
3	Hiris Gagatas	
4	Kambang Sasaki	
5	Daun Jaruju	
6	Tumpuk Manggis	
7	Bintang	
8	Kangkung Kaubakan	
9	Ombak Sinampur Karang	
10	Bayam Raja	
11	Ulat Karikit	
12	Hiris Pudak	
13	Ular Lidi	
14	Mayang Murai	
15	Naga Balimbur	
16	Ramak Sahang	
17	Gelombang	
18	Daun Katu	

Source Picture : (Rosyadi, 2017) and http://tatismp6.blogspot.com/2010/07/Sasirangan.html

In this research, repeated Sasirangan patterns were ignored so that the patterns studied were only part of the patterns shown in Table 1.

Based on Theorem 1, Theorem 2, Definition 4, the authors give the procedure to investigate the type of group on the Sasirangan patterns as follows:

1. Determine the number of rotational symmetry and the number of line symmetry of the Sasirangan pattern.
2. Give the numbering to the Sasirangan pattern.
3. Determine the permutations obtained from rotation and reflection on the line symmetry.
4. Determine the results of the operations of all permutations formed and write them in the Cayley table.
5. Determine the generator of the group based on the Cayley table.
6. Determine the type of group based on the result of the operation.

Determine the type of group based on the result of the operation.

RESULTS AND DISCUSSION

From Table 1, it is known that a Kembang Kacang pattern, Daun Jaruju pattern, Kangkung Kaubakan pattern, Mayang Murai pattern, and Naga Belimbur pattern have one rotational symmetry and non-line symmetry. If given the numbering in the patterns respectively that shown in Figure 2.

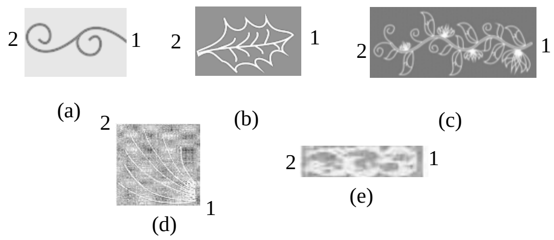


Figure 2. (a) *Kembang Kacang* Pattern, (b) *Daun Jaruju* Pattern, (c) *Kangkung Kaubakan* Pattern, (d) *Mayang Murai* Pattern, (e) *Naga Belimbur* Pattern

then it is obtained one permutation $\boldsymbol{\rho}: \boldsymbol{I_2} \to \boldsymbol{I_2}$ with $\boldsymbol{\rho} = (\mathbf{1})$. Hence, by Definition 1, it is obtained a cyclic group $\boldsymbol{C_1}$.

The *Gigi Haruan* pattern, the *Ulat Karakit* pattern, and the *Hiris Pudak* pattern are equivalent. Based on this, one of the patterns is investigated, is the *Hiris Pudak* pattern, shown in Figure 3.

Figure 3. Hiris Pudak Pattern

From Figure 3, the *Hiris Pudak* pattern has two rotational symmetry and non-line symmetry, that shown in Figure 4.

Figure 4. Rotation of the *Hiris Pudak* Pattern

Hence, obtained two permutations, namely, $\sigma: I_2 \to I_2$ and $\mu: : I_2 \to I_2$ defined respectively

$$\sigma = (1) \text{ and } \mu = (1\ 2).$$

The operation of two permutations is shown in Table 2.

Table 2. Table Cayley of Two Permutations from the *Hiris Pudak* Pattern

$\circ$	$\boldsymbol{\sigma}$	$\boldsymbol{\mu}$
σ	σ	μ
μ	μ	σ

Based on Table 2, the "Hiris Pudak" pattern form a cyclic group $\boldsymbol{C_2} = \langle \boldsymbol{\mu} \rangle = \{\boldsymbol{\sigma}, \boldsymbol{\mu}\}$.

Since The *Gigi Haruan* pattern, the *Ulat Karakit* pattern, and the *Hiris Pudak* are equivalent, the *Gigi Haruan* pattern and the *Ulat Karakit* pattern also form a cyclic group $\boldsymbol{C_2}$.

Next, the *Hiris Gagatas* pattern is investigated. It is given the numbering in its pattern that is shown in Figure 5.

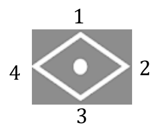


Figure 5. *Hiris Gagatas* Pattern

The *Hiris Gagatas* pattern has two rotational symmetries and two lines symmetries that are shown in Figure 6.

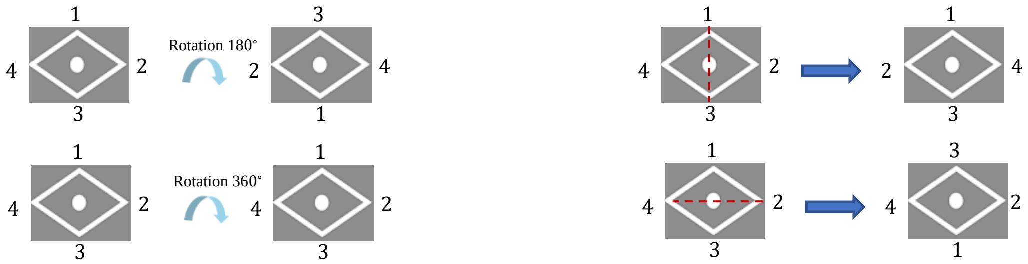


Figure 6. Rotation and Reflection of the *Hiris Gagatas* Pattern

Based on Figure 6, it is obtained that four permutations of $\boldsymbol{I_4}$ namely, $\boldsymbol{\sigma_1} = (\mathbf{1})$, $\boldsymbol{\sigma_2} = (\mathbf{1\ 3})(\mathbf{2\ 4})$, $\boldsymbol{\mu_1} = (\mathbf{2\ 4})$, and $\boldsymbol{\mu_2} = (\mathbf{1\ 3})$. The operating result of each permutation is shown in Table 3.

Table 3. Table Cayley of Two Permutations from Hiris Gagatas Pattern

$\circ$	$\boldsymbol{\sigma_1}$	$\boldsymbol{\sigma_2}$	$\boldsymbol{\mu_1}$	$\boldsymbol{\mu_2}$
σ_1	σ_1	σ_2	μ_1	μ_2
σ_2	σ_2	σ_1	μ_2	μ_1
μ_1	μ_1	μ_2	σ_1	σ_2
μ_2	μ_2	μ_1	σ_2	σ_1

Based on Table 3, it can be seen that

$$\boldsymbol{\mu_1^2 = \sigma_2^2 = (\mu_1\sigma_2)^2 = \sigma_1}.$$

So, the *Hiris Gagatas* pattern forms a dihedral group

$$\boldsymbol{D_2 = \{\langle \mu_1, \sigma_2 \rangle \mid \mu_1^2 = \sigma_2^2 = (\mu_1\sigma_2)^2 = \sigma_1\}}.$$

Furthermore, from Table 1, the *Ombak Sinampur Karang* pattern, the *Bayam Raja* pattern, the *Ramak Sahang* pattern, and the *Gelombang* pattern has two rotational symmetries and two lines symmetries that shown in Figure 7 respectively.

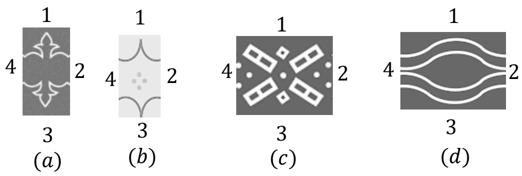


Figure 7. (a) *Ombak Sinampur Karang* Pattern, (b) *Bayam Raja* Pattern, (c) *Ramak Sahang* Pattern, and (d) *Gelombang* Pattern

Hence, the *Ombak Sinampur Karang* pattern, the *Bayam Raja* pattern, the *Ramak Sahang* pattern, and the *Gelombang* pattern also form a dihedral group $\boldsymbol{D_2}$.

For the *Daun Katu* pattern, it is given the numbering in the pattern that is shown in Figure 8.

Figure 8. *Daun Katu* Pattern

Based on Figure 8, the *Daun Katu* pattern has one rotational symmetry and one line symmetry, which is shown in Figure 9.

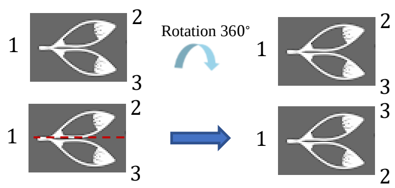


Figure 9. Rotation and Reflection of the *Daun Katu* Pattern

Based on Figure 9, obtained two permutations of $\boldsymbol{I_3}$ namely, $\boldsymbol{\rho} = (\mathbf{1})$ and $\boldsymbol{\mu} = (\mathbf{2\ \ 3})$. By operating $\boldsymbol{\rho}$ and $\boldsymbol{\mu}$ then the Cayley Table is obtained that shown in Table 4.

Table 4. Table Cayley of Two Permutations from the *Daun Katu* Pattern

$\circ$	$\boldsymbol{\rho}$	$\boldsymbol{\mu}$
ρ	ρ	μ
μ	μ	ρ

Based on Table 4, it can be seen that

$$\boldsymbol{\mu^2 = \rho^2 = (\mu\rho)^2 = \rho},$$

Such that by Definition 6 the *Daun Katu* pattern forms a dihedral group

$$\boldsymbol{D_1} = \{\langle \boldsymbol{\mu}, \boldsymbol{\rho} \rangle \mid \boldsymbol{\mu^2 = \rho^2 = (\mu\rho)^2 = \rho}\}.$$

Furthermore, since $\boldsymbol{\mu^2 = \rho}$ and $\boldsymbol{\mu\rho = \mu}$, then it can also conclude that the *Daun Katu* pattern forms $\boldsymbol{C_2} = \langle \boldsymbol{\mu} \rangle$.

Next, the *Bintang* pattern is investigated. First, investigate the *Bintang Empat* pattern. Do the numbering the *Bintang Empat* pattern as Figure 10.

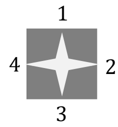


Figure 10. The *Bintang Empat* Pattern

Based on Figure 10, the *Bintang Empat* pattern has four rotational symmetries and four lines symmetries shown in Figure 11.

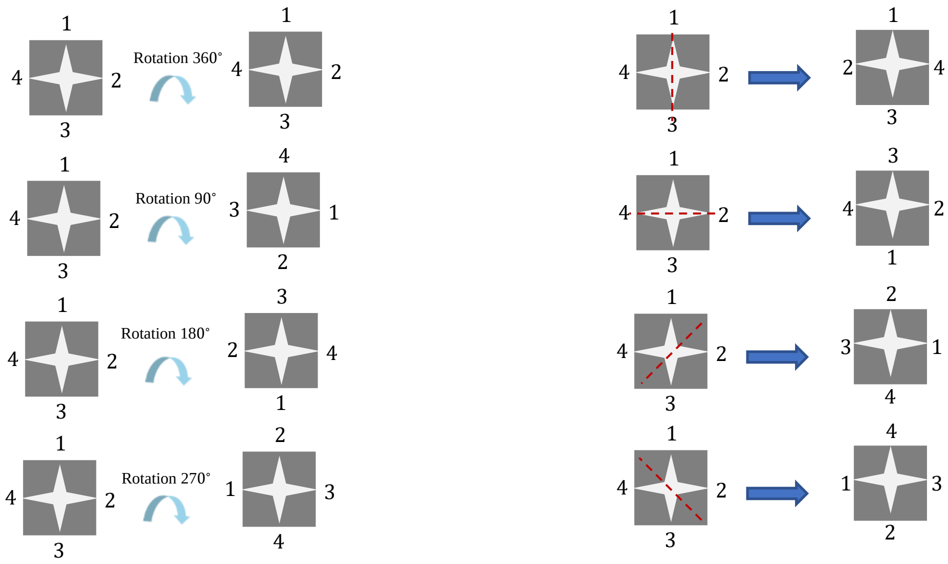


Figure 11. Rotation and Reflection of the *Bintang Empat* Pattern

Based on Figure 11, there are eight permutations of $\boldsymbol{I_4}$ namely, $\boldsymbol{\rho_1} = (\boldsymbol{1})$, $\boldsymbol{\rho_2} = (\boldsymbol{1\ 2\ 3\ 4})$, $\boldsymbol{\rho_3} = (\boldsymbol{1\ 3})(\boldsymbol{2\ 4})$, $\boldsymbol{\rho_4} = (\boldsymbol{1\ 4\ 3\ 2})$, $\boldsymbol{\mu_1} = (\boldsymbol{2\ 4})$, $\boldsymbol{\mu_2} = (\boldsymbol{1\ 3})$, $\boldsymbol{\mu_3} = (\boldsymbol{1\ 2})(\boldsymbol{3\ 4})$ and $\boldsymbol{\mu_4} = (\boldsymbol{1\ 4})(\boldsymbol{2\ 3})$. The result of the operation of all permutations is shown in Table 5.

Table 5. Cayley Table of Eight Permutations from the *Bintang Empat* Pattern

$\circ$	$\boldsymbol{\rho_1}$	$\boldsymbol{\rho_2}$	$\boldsymbol{\rho_3}$	$\boldsymbol{\rho_4}$	$\boldsymbol{\mu_1}$	$\boldsymbol{\mu_2}$	$\boldsymbol{\mu_3}$	$\boldsymbol{\mu_4}$
ρ_1	ρ_1	ρ_2	ρ_3	ρ_4	μ_1	μ_2	μ_3	μ_4
ρ_2	ρ_2	ρ_3	ρ_4	ρ_1	μ_3	μ_4	μ_2	μ_1
ρ_3	ρ_3	ρ_4	ρ_1	ρ_2	μ_2	μ_1	μ_4	μ_3
ρ_4	ρ_4	ρ_1	ρ_2	ρ_3	μ_4	μ_3	μ_1	μ_2
μ_1	μ_1	μ_4	μ_2	μ_3	ρ_1	ρ_3	ρ_4	ρ_2
μ_2	μ_2	μ_3	μ_1	μ_4	ρ_3	ρ_1	ρ_2	ρ_4
μ_3	μ_3	μ_1	μ_4	μ_2	ρ_2	ρ_4	ρ_1	ρ_3
μ_4	μ_4	μ_2	μ_3	μ_1	ρ_4	ρ_2	ρ_3	ρ_1

Based on Table 5, it can be seen that

$$\mu_2^4 = \sigma_1^2 = (\mu_2\sigma_1)^2 = \mu_1.$$

Then by the Definition 6, *Bintang Empat* pattern forms the dihedral group

$$D_4 = \{\langle \mu_2, \sigma_1 \rangle \mid \mu_2^4 = \sigma_1^2 = (\mu_2\sigma_1)^2 = \mu_1\}.$$

The *Bintang lima* pattern has five rotational symmetries and five lines symmetries. It is the same with the *Tumpuk Manggis lima* pattern with the line symmetries shown in Figure 12. Hence, in this paper, Authors only investigate the *Tumpuk Manggis lima* pattern.

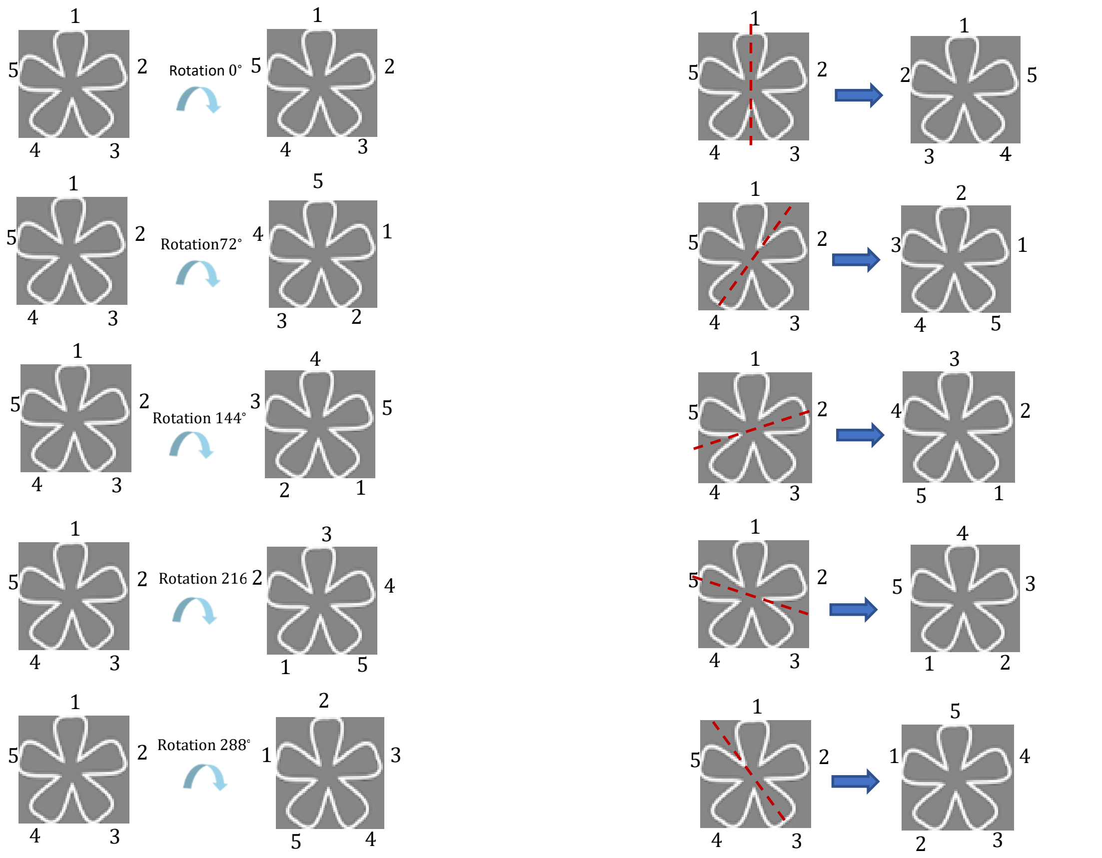


Figure 12. Rotation and Reflection of the *Tumpuk Manggis Lima* Pattern

Based on Figure 12, there are 10 permutations, they are
$\sigma_1 = (1)$, $\sigma_2 = (1\ 2\ 3\ 4\ 5)$,
$\sigma_3 = (1\ 3\ 5\ 2\ 4)$, $\sigma_4 = (1\ 4\ 2\ 5\ 3)$,
$\sigma_5 = (1\ 5\ 4\ 3\ 2)$, $\mu_1 = (2\ 5)(3\ 4)$,
$\mu_2 = (1\ 3)(4\ 5)$, $\mu_3 = (1\ 5)(2\ 4)$,
$\mu_4 = (1\ 2)(3\ 5)$, $\mu_5 = (1\ 4)(2\ 3)$.

Table 6 gives the result of all operations for all permutations.

Table 6. Cayley Table of Eight Permutations from the *Tumpuk Manggis Lima* Pattern

$\circ$	$\boldsymbol{\sigma_1}$	$\boldsymbol{\sigma_2}$	$\boldsymbol{\sigma_3}$	$\boldsymbol{\sigma_4}$	$\boldsymbol{\sigma_5}$	$\boldsymbol{\mu_1}$	$\boldsymbol{\mu_2}$	$\boldsymbol{\mu_3}$	$\boldsymbol{\mu_4}$	$\boldsymbol{\mu_5}$
σ_1	σ_1	σ_2	σ_3	σ_4	σ_5	μ_1	μ_2	μ_3	μ_4	μ_5
σ_2	σ_2	σ_3	σ_4	σ_5	σ_1	μ_4	μ_5	μ_1	μ_2	μ_3
σ_3	σ_3	σ_4	σ_5	σ_1	σ_2	μ_2	μ_3	μ_4	μ_5	μ_1
σ_4	σ_4	σ_5	σ_1	σ_2	σ_3	μ_5	μ_1	μ_2	μ_3	μ_4
σ_5	σ_5	σ_1	σ_2	σ_3	σ_4	μ_3	μ_4	μ_5	μ_1	μ_2
μ_1	μ_1	μ_3	μ_5	μ_2	μ_4	σ_1	σ_4	σ_2	σ_5	σ_3
μ_2	μ_2	μ_4	μ_1	μ_3	μ_5	σ_3	σ_1	σ_4	σ_2	σ_5
μ_3	μ_3	μ_5	μ_2	μ_4	μ_1	σ_5	σ_3	σ_1	σ_4	σ_2
μ_4	μ_4	μ_1	μ_3	μ_5	μ_2	σ_2	σ_5	σ_3	σ_1	σ_4
μ_5	μ_5	μ_2	μ_4	μ_1	μ_3	σ_4	σ_2	σ_5	σ_3	σ_1

Based on Table 6, it can be seen that

$$\boldsymbol{\sigma_2^5 = \mu_1^2 = (\sigma_2\mu_1)^2 = \sigma_1},$$

such that, by the Definition 6, the *Tumpuk Manggis Lima* pattern forms a dihedral group

$$\boldsymbol{D_5} = \{\langle\boldsymbol{\sigma_2, \mu_1}\rangle \mid \boldsymbol{\sigma_2^5 = \mu_1^2 = (\sigma_2\mu_1)^2 = \sigma_1}\}.$$

Based on the *Hiris Gagatas* pattern, the *Bintang* pattern, and the *Tumpuk Manggis* pattern, for any $\boldsymbol{n} \in \mathbb{N}$, $\boldsymbol{n \geq 2}$, if the pattern has $\boldsymbol{n}$ rotational symmetries and $\boldsymbol{n}$ lines symmetries then the pattern forms a dihedral group $\boldsymbol{D_n}$. Furthermore, since there are eight rotational symmetries and eight lines symmetries in the *Kambang Sasaki* pattern, the pattern forms a dihedral group $\boldsymbol{D_8}$ namely,

$$\boldsymbol{D_8} = \left\{\langle\boldsymbol{\sigma_2, \mu_1}\rangle \mid \boldsymbol{\sigma_2^8 = \mu_1^2 = (\sigma_2\mu_1)^2 = \sigma_1}\right\},$$

with $\boldsymbol{\sigma_1} = (\mathbf{1})$, $\boldsymbol{\sigma_2} = (\mathbf{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8})$ and $\boldsymbol{\mu_1} = (\mathbf{2\ 8})(\mathbf{3\ 7})(\mathbf{4\ 6})$.

CONCLUSIONS AND SUGGESTIONS

Based on the result and discussion, it is obtained that the pattern of Sasirangan can form a finite group. The group that formed from the pattern of Sasirangan depends on the number of rotation symmetries and the number of line symmetries of the pattern. The group that formed from a *Kembang Kacang* pattern, *Daun Jaruju* pattern, *Kangkung Kaubakan* pattern, *Mayang Murai* pattern, and *Naga Belimbur* pattern are a cyclic group C_1. Whereas the *Gigi Haruan* pattern, the *Ulat Karakit* pattern, and the *Hiris Pudak* form a cyclic group C_2.

The *Hiris Gagatas* pattern forms a dihedral group D_2 and the *Bintang Empat* pattern forms D_4. Furthermore, the *Bintang Lima* and *Tumpuk Manggis* patterns, and the *Kambang Sasaki* pattern respectively form D_5 and D_8.

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