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Enhancing mathematical proof skills in teacher education through etymology-based learning approaches

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ABSTRACT

This study aims to enhance students' understanding of mathematical proof through the etymology-based learning method. The research employs a descriptive approach to observe and analyze students' mathematical proof patterns in various contexts. Data were collected from 30 students enrolled in a master's degree program in mathematics education who demonstrated varying abilities in mathematical proof. The findings indicate that using action words or phrases during instruction can support students' ability to differentiate relevant terms and avoid misinterpretation of context. Furthermore, comparative understanding of source and target problems improves students' mathematical proof patterns in broader learning contexts. The implications of this study highlight the importance of integrating language-based and formal approaches in mathematics teaching to enhance students' mathematical proof abilities holistically.

Meningkatkan keterampilan pembuktian matematika dalam pendidikan guru melalui pendekatan pembelajaran berbasis etimologi

Kata Kunci:

Seni pembuktian matematis
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Lintasan Bahasa
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Istilah

ABSTRAK

Penelitian ini bertujuan untuk meningkatkan pemahaman siswa terhadap pembuktian matematika melalui metode pembelajaran berbasis etimologi. Penelitian ini menggunakan pendekatan deskriptif untuk mengamati dan menganalisis pola pembuktian matematika siswa dalam berbagai konteks. Data dikumpulkan dari 30 siswa yang mengikuti program pendidikan guru tingkat magister dalam bidang matematika, yang menunjukkan variasi kemampuan dalam pembuktian. Hasil penelitian menunjukkan bahwa penggunaan kata kerja atau frasa tindakan selama pengajaran dapat mendukung kemampuan siswa dalam membedakan istilah yang relevan dan menghindari kesalahan dalam memahami konteks. Selain itu, pemahaman komparatif terhadap masalah-masalah sumber dan target meningkatkan pola pembuktian siswa dalam konteks pembelajaran yang lebih luas. Implikasi dari penelitian ini menekankan pentingnya integrasi pendekatan berbasis bahasa dan pendekatan formal dalam pengajaran matematika untuk meningkatkan kemampuan pembuktian siswa secara holistik.

1. INTRODUCTION

The use of language in exploration techniques and information development can be effectively guided by two factors: the word itself and its etymological sense. These factors can provide valuable insights into the trajectory of representations, mathematical proof problems, and the overall meaning of the information [1]-[3]. One area of focus is studying students' mathematical proofs to improve the design of mathematics education. This research proposes an apsproach to investigate the development of mathematical ability by studying the process of doing mathematical proofs, taking into account etymological factors [4].

Many ideas about mathematical proof have been limited to formal and deductive forms [5]-[7]. This research on adaptive proving covers a wide range of aspects, including informal explanations and justifications and intuitive and inductive reasoning based on the etymological sense of terms [3], [8]. However, it is hard to separate significant findings in logic from those in the etymological sense [4]. For instance, the truth within a formal system is considered valid in a specific interpretation. The teaching culture is primarily based on algebra, where symbols are used to color a mathematical proof [9]. When the students face another mathematical proof representation, they tend to change or translate to algebra relationships [8].

In this research, we aim to understand better students' perspectives on using a single word or phrase to describe their work. An indexical expression is used to refer to the importance and interest in doing mathematical proofs of mathematics [5]. However, these aspects have played a crucial role in shaping the philosophical approach commonly linked to the concept of the word or sentence [10]. The role of meaning in natural languages and the relation of ontology are currently being extensively discussed and considered when proving a mathematics statement, as many students have expressed doubts. However, our efforts have yielded positive results [4].

There are many etymological meanings of ‘proving’ [2], i.e., upright, forward, go through, try, test, judge, validify, verify, and check [2]. So, proving is a process of a state and part of improving thinking [10]. In the thinking configuration, a belief is fundamental to adopt a strategy for setting a mathematical proof, i.e., to have an action of what the students do [11], [12]. Research in mathematical education has often focused on the formal and deductive forms of mathematical proof [13]. However, recent studies advocate for a broader approach that includes informal, intuitive, and inductive reasoning. For instance, Chiu [14] discusses the differential psychological processes underlying skill development in mathematics across different educational contexts. Similarly, Chiu [15] emphasizes addressing cognitive and affective issues in implementing constructivist curricula.

One way to obtain models of mathematical proof is by constructing the same statement. Suppose the structures provided are considered models of the origins of words. In that case, their representation can be seen as a model of thought since a mathematical statement is considered valid in the specific way it is used. In this case, the mathematical proof is written in a way that resembles regular written language, using a style and format that is commonly used and understood. It's doable to become skilled at mathematical proofs with some effort. So, the real question is, "How can we teach or learn the process of proving?" Based on a study, formal mathematical proof is taught until fully understood and mastered [16]. However, when it comes to proving something, there are three modes of available information for the argumentation: (a) the given information, (b) the solicited information, and (c) the feedback that needs to be presented and organized clearly and formally. Research studies have found that students' behaviors and reactions in the classroom are considered essential factors in teaching [3], [8], [17], [18]. When information and feedback are presented in words instead of symbols, it is easier for people to understand and process. These factors determine this research as we examine various words in action to demonstrate and apply mathematical concepts in real-life situations.

Research on mathematical proof methods has been conducted extensively, including contextual learning models to improve mathematical problem-solving skills [19], mathematical proof from mathematics itself [20], teachers' perspectives on mathematical proof [21], and mathematical proof in mathematics education [22]. However, none of these studies has specifically applied the etymology-based learning method to enhance mathematical proof skills. This method has the potential to help students differentiate relevant terms and avoid misleading contextual features.

This study examines the impact of applying the etymology-based learning method on mathematical proof skills at the teacher education level. Unlike previous studies, this method focuses on using action words or phrases during instruction to support students' mathematical proof abilities. Employing this language-based approach is expected to improve students' mathematical proof skills holistically.

Contribution to the literature

This research contributes to:

- Introducing an innovative approach by integrating instructional etymology into mathematics teaching.
- Providing new insights into how context-focused teaching can support deeper learning processes.
- Offering a model for how mathematics education can be tailored to meet the diverse needs of students.

2. METHOD

That research is a meta-pattern of mathematical proofs, revealing information regarding the students' mathematical proofs. The students demonstrate their mathematical abilities through mathematical proofs during mathematics tests. An underlying challenge is comparing a spectrum of mathematical proofs among 30 students. The model is both a variable (kind of word) and the varieties of mathematical ideas analyzed descriptively to explain the meta-patterns of the growth trajectories, the analogy with the longitudinal study [23].

In this research, the data was designed using etymology to provide a contextual measure. The data also offer an opportunity to creatively proof using different action words. The etymological sense emphasizes the diversity of students' mathematical proof development. The students may possess different patterns of growth trajectories and tailored mathematics educational provisions.

There are four classes of mathematical proof trajectories in school mathematics: making illustrations, taking examples, presenting pictures (or visuals), and manipulating symbols. In mathematical proof action, displays based on words are used in problems or exercises. The growth studied is in the domain of using or developing words for proving to strongly predict later mathematical proof ability.

The teaching related to mathematical proof is done by elaborating words used in mathematical proof. It focuses on mathematical proof performance based on the action word. In the learning model, this research design uses a model that integrates cognition, action, and reflection in teaching practices [1], [24].

Students' beliefs are an indispensable aspect of mathematics learning [5], [11], [25], [26]. Students' beliefs (or cultures) toward learning mathematics relate to their mathematics achievement [27], even over some time [28]. Adolescent mathematics ability perceptions or confidence can further predict mathematics ability and career self-efficacy [14], [29].

Mathematical proof can be extended and adjoined to embody properties relevant to the sense. The data is arranged in the etymological sense and presented based on its meaning or highlight. The data is interpreted using any kind of representation, text, visuals, and experience. The analysis of mathematical proof performance is done by putting it on two dimensions of a graph axis according to the etymological understanding and in meta-pattern quadrants.

Two research questions were derived from the empirical review and word trajectories. Research Question 1 focuses on identifying and inventing empirical models of mathematical proofs. Research Question 2 focuses on word trajectories in doing mathematical proofs in contextual measures. (1) How is the mathematical proof model, according to students, putting into words action in meta-pattern during master teacher education? (2) What are the trajectories of mathematics teaching and student beliefs?

The data was taken from mathematics problems posed in-class activities. It was collected from 100 students between 2018 and 2019. Thirty of the students were selected as participants. The first data are the students' performance in mathematical proof. After categorizing the data, the 30 students could be divided into the etymological sense, i.e., the intersection of the oscillation of words and their understanding of exploring the mathematical proof.

The thirty students took courses in semester 1 of the 2019-2020 academic year. They were in the master teacher educational program, and they studied senior high school mathematics. Most students already teach mathematics at various school and course levels.

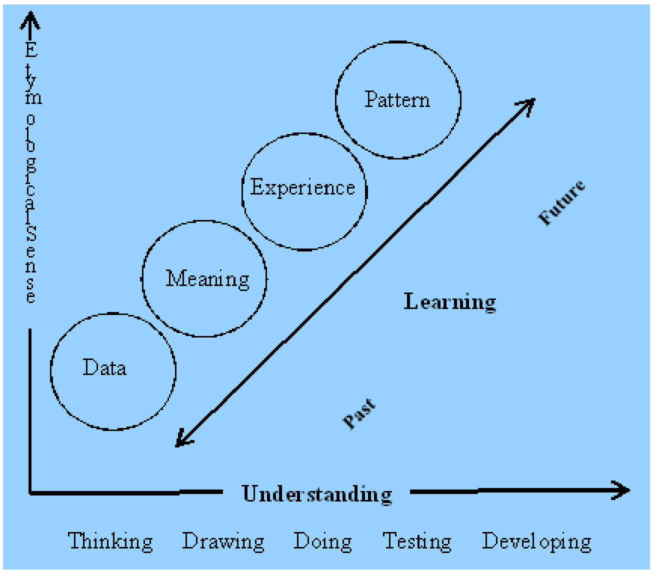

Figure 1. Data Collection Design

The research design in Figure 1 describes the oscillation model of learning and includes the students' activities. The horizontal axes showed the moving understanding of doing mathematical proof based on the mathematics representations (thinking, drawing, doing, testing, and developing). The qualitative data collected from the axes are logical connection, visual representation, the relation of the two components (doing mathematical proof), testing the relationship, and showing representation variations (developing).

On the vertical axes (etymological sense), the qualitative data are about using verb words as instruction for proving. Some of the words are to test, to try, to justify, to use, to verify, to show, and to choose (for representation).

Students need to explore the meaning and then give examples for mathematics knowledge. For example, to test that the absolute value of any real number x is more significant than or equal to 0. Because of the verb word, the students give some number for the testing.

The data was collected using the etymological sense design of meaning and mathematical proof understanding. To enhance students' understanding, the researcher started learning activities from thinking, drawing, doing, testing, and developing their performances (or may be abilities).

In doing mathematical proof, the researcher can observe students' experiences of the meaning of words and how they make a pattern in proof. The intersection of understanding and using words was observed and analyzed according to the trend for future mathematics learning, particularly in doing mathematical proof.

Figure 2 depicts the four quadrants that map the etymological sense of meaning in mathematical proof of mathematics problems (expression or statement).

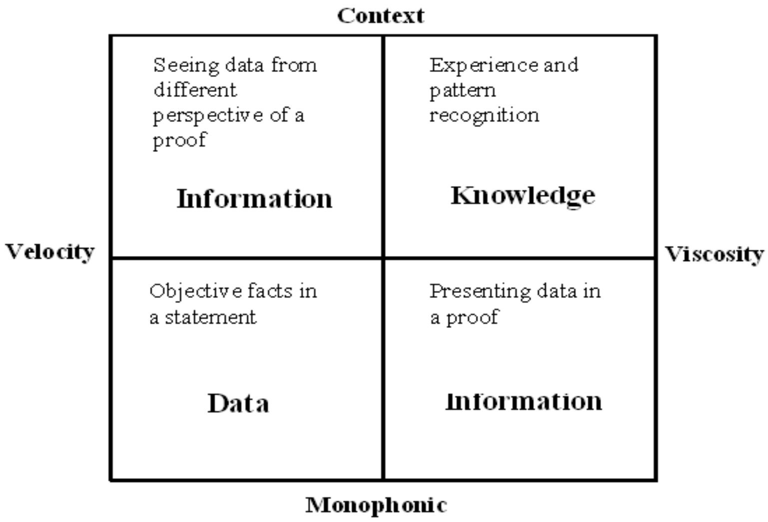

Figure 2. The Meta-Pattern Quadrant

These measures ranked the flexibility, originality, and elaboration of mathematical proofs with the following indicators: flexibility in doing mathematical proof without any formal constraints, originality in elaborating mathematical proof based on word understanding and elaborating mathematical proof using any representations. The measures of student mathematical proof were used to identify the growth of word trajectories. In addition to interpretive plausibility in mathematical proof classification, the model fit criteria were used to compare the models.

3. RESULTS AND DISCUSSION

The analysis procedure started by using words in doing a mathematical proof and ended by setting three creative aspects. The entropy values indicated the creative mathematical proof models, showing that the three models were more appropriate than the one formal model. The plausibility suggested the three models tended to have the closest fit to the empirical data and were interpretive in their growth mathematical proof trajectory patterns. Mathematics teaching had significant formal mathematical proof capacities but with a small effect in discriminating the students' performances. Compared with formal mathematical proof, the etymological sense of a mathematical proof was more advanced, with positive class interactions. The class had positive perceptions of mathematics teaching.

Students' beliefs about proving mathematics statements or expressions in different subjects show a strong desire to be taught in high school. The individuals who are less likely to become lecturers teaching at higher education are listed in decreasing order. The students were less likely to reduce their activities to prove a rigid mathematical proof than a flexible one. They worked in a typical manner, following the word's original meaning, as opposed to the conventional way, because they were involved in fewer educational activities than those focused on perception. There are variations in the difficulty level when it comes to understanding the meaning of words, and this difficulty tends to increase as we learn more. Over time, formal mathematical proofs became less common because of the way students' math abilities, the use of different language forms, and the desire to prove things interacted. The class consisted of students who had never encountered any difficulties in doing mathematical proofs.

All mathematics problems involve various concepts, such as monophonic to context and velocity to viscosity. Students' general knowledge is similar, but their ability to apply that knowledge to solve specific problems may vary. For instance, when trying to understand and connect existing knowledge with unfamiliar information. That's an issue with formal mathematical proof. When looking at a mathematical proof from different perspectives, the motivation to write down a mathematical proof primarily comes from the need to be formal. The problem was solved by using numbers and operations to determine the velocity. When they receive a statement or problem, they analyze the objective facts and consider how they relate to the context.

The important thing is that when students passed through the Monophonic-Context axis, they presented data from a problem by examining the pattern. When faced with a complex problem, people often use numbers or equations to help them understand the situation. This process involves transforming a graph, a type of action that can be taken. For instance, the graph of the absolute value function $|2x-1|$ can be represented as $(2x – 1)$ for values of x greater than or equal to ½, or as the negative of $(2x – 1)$ or $(1 – 2x)$ for values of x less than or equal to ½. When dealing with negative numbers, there is a helpful visual representation. It involves moving from zero towards a number line's left and right. It's important to understand that zero is the foundation of the absolute concept, a way of thinking that involves flexibility and adaptability.

The viscosity axis, located to the right of the monophonic context, serves as a foundation for conducting mathematical proofs. The student's understanding of the quadrant promotes critical thinking regarding mathematical proofs. An example of this line of thinking is that the data in a mathematics problem is presented in a way that is meant to be persuasive. It seems that some testing has been done to determine the truth of something. The solution set for the absolute value of $|2x-1|$ is $y \geq 0$, which shows the students that these are the values.

The researcher discovered the relationship between words (or phrases) and mathematical symbols (or notations). The problem is primarily focused on the words rather than the symbols. Symbolically representing phrases like' at least,' 'more than," or at a time, least than or equal to' can be challenging. When a symbol is transformed into a word, it loses its richness and becomes either less than or greater than. That's an issue with learning mathematics.

Mathematical proofs can help us understand the origins and true meanings of concepts. They show us how ideas evolve and become more complex as we find better solutions. The percentages align with those of undergraduate and high school teachers. That means there are significant differences in performance, and it has been found that students and teachers can understand mathematical proofs.

Extending mathematical proof has allowed us to discover and understand unique phenomena. The process of developing a formal mathematical proof usually remains consistent and steady. The approach primarily used was deductive, relying on abstraction. The performance has decreased, putting it on par with that of lecturers and numerous textbooks. However, we found something quite intriguing as we looked at how students perform regarding their ability to generate original ideas.

The performances of doing mathematical proofs showed a noticeable improvement over time, with the initial performances demonstrating a more detailed and adaptable approach and later performances showing a slight but consistent improvement (i.e., generally positive slopes). That is a very interesting pattern in the way mathematical proofs are performed. It is also consistent with finding meaning in words and is relatively stable. When we look at the different measures, we can see that the learning activities (precisely the experiment method), the lecturer's notes, teaching methods, and textbooks can help students who don't often speak or use everyday language to demonstrate their understanding.

One interesting thing is that the students seem to struggle with their ability to do mathematical proofs, which is something that hasn't been seen in other studies. The results of the meta-pattern analysis explain the reasons. The students who excel in formal mathematical proof positively perceive mathematics teaching. They are motivated to pursue higher education, especially in proving. As a result, they may choose to disengage from certain activities to focus on the deeper meaning of words. Their experiences are measured using various symbols and logic, but they do not perform better than the contextual measures of a heightened sense of meaning. Future research should validate these two unique phenomena.

The results reveal meaningful information about students' beliefs, mathematics teaching, and textbook preferences about their performance in mathematical proof. The more formal and rigorous a mathematical proof is, the fewer activities there are in proving based on textbooks or teaching methods influenced by the culture of the lecturer. The student's beliefs are still based on formal mathematical proof, and they have discovered that they tend to present mathematical proof oscillating.

Students who excel in mathematical proof tend to appreciate high-quality teaching in mathematics, including lectures, interactive learning, and textbooks, instead of focusing solely on the literal meaning of the subject. The performance indicates that when teaching mathematical proof, it is essential to connect it to the activities of the students to gradually guide them toward understanding and proving. Plotting points on grid paper involves using symbols or visuals to represent data. Mathematical proof originated from equations, while its meaning evolved from real-world observations and the construction of various concepts before being formalized. That means having positive thoughts and beliefs, like building a path towards success. However, it's important to note that the study's results do not establish a cause-and-effect relationship. Therefore, it is necessary to explore the reasons behind these findings from various angles. It is also worth considering the possibility of a reciprocal influence between mathematics teaching and the ability to understand mathematical proofs.

The results suggest that high-achieving students may benefit from better mathematics teaching, including a deeper understanding of the meaning behind mathematical concepts (particularly in school maths). When we look at it from the other perspective, students who are good at math proofs may be more likely to recognize and appreciate high-quality math teaching rather than being influenced by other factors like textbooks, personal beliefs, or formal presentations. Simply put, what students think about doing mathematical proofs doesn't matter when it comes to how math is taught. Previous research has shown that there isn't a clear connection between teaching methods and students' math abilities.

For future research, it is suggested to consider the limitations of this study and focus on selecting empirical contextual measures that can contribute to the development of a mathematics learning culture. The measures, however, were primarily designed based on the origins of words and students' beliefs in the importance of mathematical proof and how it relates to learning mathematics. For instance, mathematics problems can be presented in a language-based format rather than solely using symbols. In the future, it would benefit researchers to expand on their findings by focussing on mathematics education or conducting experimental studies within scientific cultures. Some of the contextual measures were measured in a discrete or dichotomous manner. When people report their opinions using a Likert scale, it may not always give enough detailed information for a thorough discussion.

The meaning behind proving school mathematics (vertical-axis) was to represent a mathematical proof. These consist of data, significance, learning experiences, and patterns. The data presented in a mathematics problem consists of information to consider and analyze. That's just a phrase and it's not sufficient for conducting a mathematical proof. For instance, let's consider a test: Let's consider angle θ. On the starting side of the angle, there is a point called P that is located one unit away from the vertex. If θ is greater than or equal to 0, we can define s as the arc length traced by point P as the initial side rotates through θ.

On the other hand, if θ is negative, then the arc length s will also be negative. In this case, the radian measure of θ is equal to s. The students ask for a description and an equation in degrees and radians.

The term 'rotation' also has an etymological sense of meaning. Describing the situation accurately is a challenge. Another term that can be used is 'negative or positive,' which resembles a numerical value. However, in that particular context, the words hold meaning beyond mere numerical values and indicate a specific direction. Regarding mathematical proofs, it's important to understand that words or phrases carry specific mathematical meanings. The learning process plays a crucial role in teaching this subject. The issue here is that the two words have different unit measures.

The ability to perform mathematical proofs was considered an important aspect of a student's mathematical skills and, therefore, an indicator of effective teaching. It seems that doing mathematical proof analysis was necessary. In terms of hierarchy, the word "data" 's initial meaning refers to the information being presented as evidence. A range of students' levels of doing mathematical proof has been developed. The growth lies in the act of proving and the capacity to provide evidence. When people prove a statement, they aim to understand better how to prove it by identifying patterns.

The findings indicate that the systems of elementary logic should be seen as the practical decision of accepting the truth of a statement and assuming it to be correct. A more general concept than validity is the relationship between a set of sentences and a single sentence that holds, often referred to as the statement's implication or common sense. It seems like there is a need for an informal system of truth. That means simplifying a formal representation to a specific example and considering how well it captures the situation.

Knowledge can be understood objectively by expressing mathematical proofs and solving problems. The data is unique to the current class environment designed by the researcher. It's difficult to keep up with the fast-paced data and try to prove and understand it simultaneously. The context involves intertwining or linking words to function within a mathematical proof. It influences how students perceive and understand the meaning. Students receive information and knowledge in a variety of ways and contexts. The students pay attention to the contextual clues connected to each meaning to understand or communicate the intended message. For example, when proving a symbolic implication statement like $0<x<1 \rightarrow 0<x^2<1$, the students use their judgment, make determinations, evaluate the situation, and consider the truth.

Patterns and context have a close relationship. The pattern tends to create its own context rather than being as dependent on context as information. The context is usually provided explicitly, especially when it can be easily recognized, such as in a textbook. To understand the hidden context, it is necessary to identify the relevant information about the problem. Verbalizing properly can often be quite challenging and may not always be consistent. Understanding a mathematical proof can often be the most challenging attribute.

There are a few clear and consistent findings that are also backed up by some of the results. The findings reveal that students are more successful when they engage in the process of constructing mathematical proofs. Additionally, they demonstrate a significant ability to develop mathematics teaching and learning programs tailored to meet students' specific needs. The researcher, in this case, had a good understanding of the participants and customized their programs to meet their specific learning needs and level of mathematical understanding.

The findings challenge some commonly held beliefs about the school curriculum's rigid structure based on individual subjects. There was no evidence to suggest that effective teaching and learning in school mathematics were incompatible. Over the past thirty years, there have been significant changes in how schools manage their internal structures and career paths. These changes have sometimes hurt the status and responsibilities of individuals in certain sectors, particularly in other positions of responsibility in school mathematics.

The research findings provide a compelling reason to reevaluate the current inclination toward the etymological meaning, particularly in judging, trying, verifying, determining, and testing. The immediate work seems to have a significant impact on teaching and learning. Overall, the studies suggest that it might be worthwhile to consider the importance of the historical origins of mathematical proof and whether the current methods of training educators adequately prepare them for the transition into the role of Subject Coordinator.

The combination of these levels represents a teaching and learning model for understanding mathematical proof concepts. This model has been evaluated using nine performance variables, as shown in Table 1.

Table 1. The Level of Teaching and Learning of Etymological Mathematical Proof

Level of performance	Goals	Design	Management
Class	Strategy	Structure	Resources
Process	Developing	Improving	Reengineering
Individual	Coaching	Intervention	Elaborating

Managing the nine performance variables will contribute to teaching and learning mathematics quality by offering diverse opportunities to explore the etymological origins and discover various mathematical proof representations. There is a sense of experience. The activity involves making sense and providing proof. Furthermore, the reasoning accounts for the student's performance are influenced by dissociation.

The student's engagement in mathematical proof should be a concern when evaluating mathematics education, particularly in learning and teaching mathematics. For example, I discovered that understanding words' origins can positively impact learning and academic success. How people feel about mathematics is shaped by their early learning experiences, which affects how students perceive and understand mathematics as they grow older. Unfortunately, students still rely on educators, friends, or other sources to imitate when learning mathematics.

The original meaning of "meaning" in mathematical proof focuses on how words or phrases are connected to a student's ability to understand and develop mathematical proofs. When considering the growth context, we explore how the meaning of words evolves and the significant impact that elaborating on ideas has on mathematical proof. It is important to understand how mathematical proofs are developed and the patterns that emerge in the process. The growth trajectories are shown in Table 2.

Table 2. Trajectories Performances Mathematical Proof

Etymological Sense	Understanding	Trajectories
Words	Thinking	Data
Phrase	Drawing	Meaning
Sentence	Doing	Experience
Logical connection	Testing	Pattern

The original concept of mathematical proof involves using ordinary sentences that represent numbers and operations as symbolic expressions with corresponding interpretations. It is possible to use a word when doing a mathematical proof, such as referring to the period of a trigonometry function as a transformation. Students think about it based on data and visual representations of the graph. However, it is necessary to use grid paper when it comes to demonstrating mathematical proofs.

Simply put, you must express it through equations or visually when it comes to proving something in mathematics. The purpose of the trajectory is to go beyond mere symbolic expression and acquire deeper meaning. To understand the meaning of a graphic, students should start by avoiding algebraic thinking. For instance, the keyword is transformation. When students attempt to establish a symbolic relationship, they must test certain points and use a matrix. The process of causation is not always limited to formal mathematical proofs but can also involve simple sentences and their combination to guide a desirable course of action. If a mathematical expression or statement depends on the dimension, it means that it is related to the structure of an object or system. As students, we have the ability to perceive or understand a truth, not just based on its definition, but through the way it is meant to be understood. A short sentence in mathematics is considered true only if it aligns with our intuitive understanding (known as a fact). However, in the etymological sense, truth refers to the ability to provide a clear and specific interpretation.

A logical connection is also a sense that can be traced back to its etymology. It appears that mathematical discoveries are used to clarify concepts related to logical truth. One way to extract trajectories from the given words is by constructing their meaning. When a sentence's structure is copied, its etymology becomes a powerful and intuitive idea, as every sentence holds truth in all contexts. For instance, let's consider the idea of a variable. Looking at its etymological sense helps us understand whether it is limited or unlimited.

The concept of completeness in a mathematical proof can be further explored straightforwardly. For instance, if a sentence talks about equality, another sentence can be added to it that includes some characteristics of a relationship that are related to the first sentence. When combined, two sentences can be treated as a single sentence in a formal mathematical proof. That is a model in the sense of its meaning, and we can understand the completeness of a mathematical proof by looking at various representations.

The findings of this research offer valuable insights for teaching and learning, suggesting that (1) it is beneficial to begin developing mathematical proof skills by understanding the etymological sense of meaning; (2) students can enhance their mathematical proof abilities by thoroughly examining their sentences; (3) it is advantageous to start with sentences or words and gradually progress towards more advanced mathematical proof development. The students exhibit slightly varied patterns in their trajectories of mathematical proof ability. Some people go through challenging experiences not just to prove themselves, while others strive to improve their performance and personal growth. This is about the students and aims to promote a better understanding of mathematical proof concepts. The students are eager to demonstrate their knowledge formally, as they value the cultural and educational significance of their previous experiences.

Learning mathematical proof often significantly impacts students, as it is influenced by the exercises presented in textbooks and tailored to their specific needs. The students argued that to demonstrate their literacy, they require a deep understanding of the origins and meanings of words, which they can acquire through a gradual process leading to a formal presentation. Both students and lecturers have encountered difficulties with the traditional teaching materials mentioned in the text. The material should use easier language for students and teachers to read and understand.

The students mentioned requiring language terms for learning and demonstrating their knowledge. They should provide evidence using mathematical concepts and terminology. That's why they need some kind of etymological material to expand the representation. Based on a needs analysis, it was determined that most students require practical teaching materials and improvement in their language skills. The exercise you mentioned is often used to demonstrate concepts and typically involves practicing in a classroom setting with various tasks. The exercise is designed to help students improve their ability to prove things. When it comes to doing mathematical proofs, most students find them equally challenging. However, it is important to practice and study them in class.

The focus of the evaluation will be on demonstrating reading comprehension tailored to the specific learning context. Regarding the type of student materials, approximately 20% require formal mathematical proof, while the remaining 80% involve evaluating practical applications. The test techniques involve evaluating individuals and groups and require different types of evaluations, such as formal logic and etymology.

To support the students' beliefs, it is crucial to have a plan in place when starting a mathematical proof. When you're making an argument, it's helpful to have a list of action words. When engaging in a practical activity, it is important to understand the meaning of geometric shapes in terms of their origins. When tabling data, we are essentially trying to establish a relationship, which can be challenging. Therefore, based on the available information, we must follow certain steps to determine this relationship. Also, it's important to note that an unknown variable isn't just used to find an answer but can also represent data in a symbol or notation. It is implied in the data and necessary to consider the condition related to a similar problem. So, I'm thinking about the etymology of something in general.

This study confirms previous research that teaching maths using etymology-based learning can help students improve their understanding and ability to solve mathematical proofs. A recent study conducted by Vankúš has found that using contextual learning approaches, such as game-based learning or inquiry-based learning, can positively impact students' motivation, engagement, and attitudes toward mathematics [30]. Our research results support these findings, showing that using action words in etymology-based learning improves students' understanding and mastery of mathematical concepts. Therefore, this study supports the idea that including contextual approaches in math instruction is important for comprehensively enhancing students' mathematical proof skills.

On the other hand, it's important to remember that this study has a few limitations. Firstly, it's important to note that the number of students included in the sample size is relatively small, consisting of only 30 individuals. The small number of participants in this study might not accurately reflect the larger population, so the results may not apply to all educational settings in mathematics. Furthermore, it is important to note that the sample used in this study has limited demographic variation. As a result, the findings may not accurately represent the diverse backgrounds of various students.

Furthermore, this study employs a descriptive methodological approach. While this approach offers valuable insights, it may be less reliable in establishing causal relationships between the investigated variables. In addition, this study did not have strict controls to compare the effectiveness of the etymology-based learning method to other learning methods.

To make future research more reliable, it is suggested that the sample size be increased to enhance the applicability of the results. It would be beneficial to conduct research with a larger and more diverse group of participants to understand better how effective the etymology-based learning method is. Moreover, by incorporating a wider range of research methods, such as experimental quantitative approaches and longitudinal studies, we can obtain more robust evidence regarding the lasting effects of this method. It would be interesting to explore how technology, like educational software or interactive learning applications, can support contextual learning. Additionally, it would be valuable to conduct further research that delves into a detailed examination of how particular action words impact the process of mathematical proof. This could be achieved through experimental studies comparing groups utilizing various action words in instructional practices.

4. CONCLUSION

The study found that the etymology-based learning method effectively enhances students' mathematical proof abilities, with results indicating that using action words during instruction can help students differentiate relevant terms and avoid misleading contextual features. Furthermore, a comparative understanding of source and target problems has improved students' mathematical proof patterns within broader learning contexts. The implications of these findings emphasize the importance of integrating language-based and formal approaches in mathematics teaching to holistically enhance students' mathematical proof abilities, making it an effective alternative for more contextual and meaningful mathematics instruction.

AUTHOR CONTRIBUTION STATEMENT

MR contributed to designing the research framework, drafting the initial manuscript, leading the research coordination, formulating the research questions, and analyzing and interpreting the data. S played a role in the research design and methodology, developing the qualitative analysis protocol, providing critical revisions to the manuscript, and supporting the data collection process. SP was responsible for collecting data, ensuring data accuracy, preparing and implementing the research instruments, and participating in the manuscript drafting and review stages.

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